The "Pioneer effect" as a manifestation of the cosmic expansion in the solar system

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Abstract

It is proposed that the recently reported anomalous acceleration acting on the Pioneers spacecrafts should be a consequence of the existence of some local curvature in light geodesics when using the coordinate speed of light in an expanding spacetime. This suggests that the Pioneer effect is nothing else but the detection of cosmological expansion in the solar system.

Pioneer effect. A careful analysis of orbital data from Pioneer 10/11 spacecrafts has been reported[1] which indicates the existence of a very weak acceleration - approximately $\kappa \simeq 8.5 \cdot 10^{-8} cm/s^2$ - directed toward the Sun. The most conservative (or less adventurous) hypothesis is that the Pioneer effect does not entail new physics and that the detected misfit must be due to some sophisticated (technological) reason having to do with the spacecraft configuration. However, the analysis in[1] seems to have ruled out many (perhaps all) of such technical reasons and the authors even claim having taken into account the accepted values of the errors in the planetary ephemeris, Earth's orientation, precesion, and nutation.

Thus, in principle, a new effect seems to have unexpectly entered the phenomenology of physics. On the other hand, if such an effect really exists (i.e., it can not be eliminated by data reanalysis) it would represent a violation of Birkhoff's theorem in general relativity for no constant acceleration at all is predicted by the Schwarzschild solution. The nature of the effect is still far from being clarified. One of its surprising features, as pointed out in [1], is that it does not affect the planets (since no cumulative precession is observed in their trajectories) but only to objects with masses similar to that of spacecrafts (an apparently, strong violation of the equivalence principle!)

Apart from their masses, the only sensible difference between planets and spacecrafts is the nature of their corresponding orbits, i.e., their relative motion with respect to the Sun. That is why the effect might be originated from some unexpected correction to the way we compute relative motions in the solar system.

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An aspect that has not been analyzed in planetary ephemeris is the observed difference of the expansion of the universe for nearby points (the Sun and the planets, say). This could amount to correcting the relative accelerations as computed as a given time of cosmological expansion.

Let us start by considering trajectories in a FRW metric

$$ds^2 = -c^2dt^2 + \chi(t)^2dr^2 \ , \tag{1}$$

where, taking our units of space and time at the cosmological time t_1, we can write,

$$\chi(t) \simeq (\frac{t}{t_1})^p \ , \tag{2}$$

where $p < 1$ is a constant depending on the density of the universe, and t_1 is the local "cosmic time".

Light geodesics satisfy

$$dl \equiv cdt = \chi dr \ , \tag{3}$$

where dl is the lenght on the null cone.

On the other hand, we should be able to write our physical laws in such a way that the expansion of space time be scaled out. This requires using the radial function,

$$r_* \equiv \chi r \ , \tag{4}$$

the metric then becoming

$$ds^2 = -(1 - \frac{r_*^2H^2}{c^2})c^2dt^2 + dr_*^2 - 2r_*Hdr_*dt \ . \tag{5}$$

where the local Hubble parameter is

$$H = \frac{d}{dt}\log(\chi) \ . \tag{6}$$

Since these are not syncronous coordinates (for $g_{0r_*} \neq 0$), we define the radial vector

$$\vec{g} = \frac{r_*H/c}{1 - r_*^2H^2/c^2}\vec{r}_1 \ , \tag{7}$$

so that the space like element, as measured by some local observer, is the embedded three dimensional metric within the global space time in this manifold (see e.g. [2])

$$dl_*^2 = (g_{r_*r_*} - g_{00}g^2)dr_*^2 = \frac{dr_*^2}{1 - r_*^2H^2/c^2} \ . \tag{8}$$

Me may now compare the lenght, l_* in the locally scaled coordinates, with l on the light cone. In order to do this, notice that one might also have obtained, after (3) and (4), the following equation for the null cone,

$$dl = dr_*(l_*) - r_*(l_*)H\frac{dl}{c} \ , \tag{9}$$

whose solution - using (8), and noting that $\dot{H} \sim O(H^2)$- is

$$l = \frac{c}{H} \log\{1 + \sin(\frac{Hl_*}{c})\} \simeq l_* - \frac{Hl_*^2}{2c} + O(H^2) \ . \tag{10}$$

This represents the measure of the space time curvature on the local null cone.

Now, let us return to the original problem of the detected misfit between the calculated and the measured position in the spacecrafts. First, we must take into account that Sun's gravitational interaction takes place on points on the light cone. This means that the real position of any particle with respect to the center of forces is l, and, after Equation (10), we realize that this quantity differs from the expected distance, l_*, (as computed upon using the local frame) just in a systematic "bias" consisting on an effective residual acceleration directed toward the center of coordinates; its constant value is

$$\kappa = Hc \ . \tag{11}$$

This is the acceleration observed in Pioneer 10/11 spacecrafts. From the value reported in [1] we get a value for Hubble parameter

$$H = \frac{\kappa}{c} \simeq 85 km/s \cdot Mpc \ . \tag{12}$$

The remarkable fact is that (10) is a function of the radius not of the time. This means that a periodic orbit does not experience the systematic bias but only a very small correction

$$-\frac{H}{2c} l_*^2 \sim 10^{-4} \text{meters} \ , \tag{13}$$

which is not detectable.

The observed fall to the center of forces does not entail cumulative effects in the orbital parameters since the result is frame dependent. This is easily seen. As stated above, the parameter p is a function of the density of the Universe, μ. Let us here make the approximation $\mu \simeq 0$, i.e., $p = 1 - \delta$, $\delta \ll 1$, -recall that δ is proportional to the global density of the universe and we must neglect its contribution in comparation with the local density of matter in the solar system. Since, on the other hand, the curvature of the light cone only depends upon the time development of the manifold, then, in order to prevent the existence of observable cumulative precesion on the orbits, we require some new set of space and *time* coordinates. We can select, for instance, the following transformation which, for $p = 1$, relates the Lorentzian metric with the Milne ($\chi = Ht$) space time:

$$dt = \frac{1}{(c^2\tau^2 - \tilde{r}^2)^{1/2}} [c\tau d\tau - \frac{\tilde{r}}{c} d\tilde{r}] \ , \tag{14}$$

$$dr = \frac{pc^2}{H(c^2\tau^2 - \tilde{r}^2)} [\tau d\tilde{r} - \tilde{r} d\tau] \ . \tag{15}$$

Using these transformations, Equation (1) becomes

$$ds^2 \to -c^2 d\tau^2 + d\tilde{r}^2 + \frac{2c^2\delta}{\tilde{r}^2 - c^2\tau^2} \log[eH(\tau^2 - \frac{\tilde{r}^2}{c^2})^{1/2}]\{\tilde{r}d\tau - \tau d\tilde{r}\}^2 \ . \tag{16}$$

For $\delta \simeq 0$, it corresponds to the Minkowski space Now, $ds^2 = 0$ leads to the differential equation

$$\frac{d\tilde{r}}{d\tau} \simeq c\{1 + \delta \frac{c\tau - \tilde{r}}{c\tau + \tilde{r}} \log[eH(\tau^2 - \frac{\tilde{r}^2}{c^2})^{1/2}]\} \ , \tag{17}$$

whose solution is fairly simple, $\tilde{r} = c\tau + O(H^2)$ - independently of the value of H. This means that the effect can be exactly removed in these coordinates. This satisfactory fact agrees with the consequences of Birkhoff's theorem.

Conclusions. The special features of the physical phenomena, including those properties that correspond to the motion of the bodies, become different in different systems of coordinates. This fact is illustrated, for instance, in the old Foucault pendulum experiment. There, the motion of the pendulum experiences the effect of the Earth based reference system -being not an inertial frame relatively to the "distant stars". We have learnt, from the previous arguments, that *Pioneer effect* is a kind of a new cosmological Foucault experiment, the solar system based coordinates, being not the true inertial frame with respect to the expansion of the universe, mimics the role that the rotating Earth plays in Foucault's experiment.

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